

# RECENT RESULTS IN THE CENTER VORTEX MODEL FOR THE INFRARED SECTOR OF YANG-MILLS THEORY<sup>a</sup>

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A model for the infrared sector of  $SU(2)$  Yang-Mills theory, based on magnetic vortices represented by (closed) random surfaces, is presented. The model quantitatively describes both confinement and the topological aspects of Yang-Mills theory. Details (including an adequate list of references) can be found in the e-prints hep-lat/9912003 and hep-lat/0004013, both to appear in Nucl. Phys. B.

Diverse nonperturbative effects characterize strong interaction physics. Color charge is confined, chiral symmetry is spontaneously broken, and the axial  $U(1)$  part of the flavor symmetry exhibits an anomaly. Various model explanations for these phenomena have been advanced; to name but two widely accepted ones, the dual superconductor mechanism of confinement, and instanton models, which describe the  $U_A(1)$  anomaly and spontaneous chiral symmetry breaking. However, no clear picture has emerged which comprehensively describes infrared strong interaction physics within one common framework. The vortex model presented here <sup>1,2</sup> aims to bridge this gap. On the basis of a simple effective dynamics, it simultaneously reproduces the confinement properties of  $SU(2)$  Yang-Mills theory (including the finite-temperature deconfinement transition), as well as the topological susceptibility, which encodes the  $U_A(1)$  anomaly. Remarks on the chiral condensate, an important point of investigation which has not yet been carried out, will be made in closing.

Center vortices are closed chromomagnetic flux lines in three-dimensional space; thus, they are described by closed two-dimensional world-surfaces in four-dimensional space-time. In the  $SU(2)$  case, their magnetic flux is quantized such that they modify any Wilson loop by a phase factor  $(-1)$  when they pierce an area spanned by the loop. To arrive at a tractable vortex model, it is

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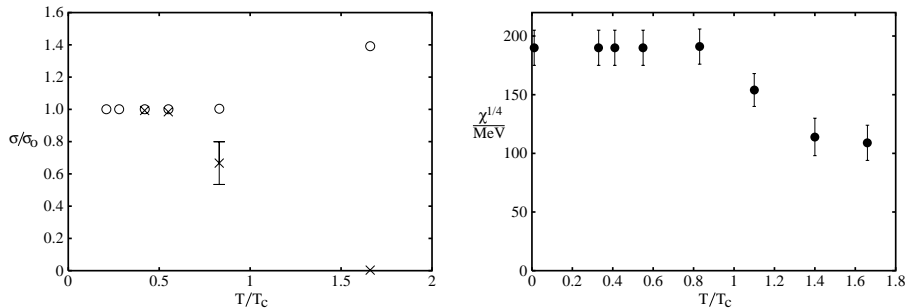


Figure 1: Observables in the random vortex surface model on  $16^3 \times N_t$  lattices, with  $c = 0.24$ , as a function of temperature. Left: String tension between static color sources (crosses) and spatial string tension (circles). Whereas the quantitative behavior of the static quark string tension has largely been fitted using the freedom in the choice of  $c$  (see text), the spatial string tension  $\sigma_s$  is predicted. In the deconfined regime, it begins to rise with temperature; the value  $\sigma_s(T = 1.67 T_C) = 1.39 \sigma_0$  corresponds to within 1% with the value measured in full  $SU(2)$  Yang-Mills theory.<sup>5</sup> Right: (Fourth root of) the topological susceptibility; also this result is quantitatively compatible with measurements in full Yang-Mills theory.<sup>6</sup>

useful to compose the vortex world-surfaces out of plaquettes on a hypercubic lattice. The spacing of this lattice is a fixed physical quantity (related to a thickness of the vortex fluxes), and represents the ultraviolet cutoff inherent in any infrared effective framework. The model vortex surfaces are regarded as random surfaces, and an ensemble of them is generated using Monte Carlo methods. The corresponding weight function penalizes curvature by associating an action increment  $c$  with every instance of two plaquettes which are part of a vortex surface, but which do not lie in the same plane, sharing a link (note that several such pairs of plaquettes can occur for any given link).

Via the definition given above, Wilson loops (and, in complete analogy, Polyakov loop correlators) can be evaluated in the vortex ensemble, and string tensions extracted. For sufficiently small curvature coefficient  $c$ , one finds a confined phase (non-zero string tension) at low temperatures, and a transition to a high-temperature deconfined phase. For  $c = 0.24$ , the  $SU(2)$  Yang-Mills relation between the deconfinement temperature and the zero-temperature string tension,  $T_C/\sqrt{\sigma_0} = 0.69$ , is reproduced. When furthermore setting  $\sigma_0 = (440 \text{ MeV})^2$  to fix the scale, measurement of  $\sigma_0 a^2$  yields the lattice spacing  $a = 0.39 \text{ fm}$ . The full temperature dependence of the string tensions is displayed in Fig. 1 (left). Note that the confined and deconfined phases can alternatively be characterized by certain percolation properties of the vortices.<sup>1,3</sup>

Complementarily, also the topological properties of the Yang-Mills ensem-

ble encode important nonperturbative effects. The topological charge  $Q$  of a vortex surface configuration is carried by its singular points,<sup>4,2</sup> i.e. points at which the set of tangent vectors to the surface configuration spans all four space-time directions (a simple example are surface self-intersection points). Since a vortex surface carries a field strength characterized by a nonvanishing tensor component associated with the two space-time directions locally orthogonal to the surface,<sup>4</sup> these singular points are precisely the points at which the topological charge density  $\epsilon_{\mu\nu\lambda\tau} \text{Tr } F_{\mu\nu} F_{\lambda\tau}$  is non-vanishing. In practice, implementing this result for the hypercubic lattice surfaces used in the present model involves resolving ambiguities<sup>2</sup> reminiscent of those contained in lattice Yang-Mills link configurations. The resulting topological susceptibility  $\chi = \langle Q^2 \rangle / V$ , where  $V$  denotes the space-time volume under consideration, is exhibited in Fig. 1 (right) as a function of temperature. Taken together, the measurements in Fig. 1 show that the vortex model provides, within one common framework, a quantitative description not only of the confinement properties, but also of the topological properties of the  $SU(2)$  Yang-Mills ensemble.

One obvious generalization of the present work is the treatment of  $SU(3)$  color. Also, the coupling of the vortex degrees of freedom to quarks must be investigated, e.g. whether the correct chiral condensate is induced in the vortex background. In this respect, the vortex picture has an important advantage to offer. It is possible to associate any arbitrary vortex surface with a continuum gauge field,<sup>4</sup> including the surfaces generated within the present model. As a consequence, the Dirac operator, encoding quark propagation, can be constructed directly in the continuum, and some of the difficulties associated with lattice Dirac operators, such as fermion species doubling, may be avoidable.

## Acknowledgments

M.E. and H.R. acknowledge DFG financial support under grants En 415/1-1 and Re 856/4-1, respectively. M.F. is supported by Fonds zur Förderung der Wissenschaftlichen Forschung under P11387-PHY.

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